The "Columbo Teacher"

Rethinking Mathematics Education in the Age of Artificial Intelligence

January 2026

Article written with the assistance of artificial intelligence...

Introduction: A Silent Revolution

The history of education is punctuated by technological ruptures that don't merely add tools but sometimes redefine the very nature of the knowledge being taught. The arrival of graphing calculators, followed by dynamic geometry software, has already profoundly modified our practices. Today, generative artificial intelligence constitutes a new rupture—perhaps the most profound since the invention of the printing press.

In France, according to a survey conducted by INRIA in 2024, nearly 90% of tenth-grade students (ages 15-16) report having already used generative AI to help with their homework. Faced with this reality, two attitudes clash: the temptation of outright prohibition, or that of thoughtful integration.

What Research Teaches Us

Aurélie Bourdais (University of Lyon, 2021) documented high school students' practices with online translators. Her finding: *nearly 70% of teachers prohibit these tools*, yet students use them massively—in secret. Prohibition is ineffective. The solution? Train rather than forbid.

This article defends a third path: that of transforming our pedagogical stance, which we present as the shift from the "Sherlock Holmes" model to the "Columbo" model. This metaphor, far from being anecdotal, represents in our view a profound paradigm shift.

1. Lessons from the Past: When Technology Shifts the Stakes

1.1 The Revealing Example of Variation Tables

Before the advent of graphing calculators, variation tables represented the culmination of a long analytical process. They were the indispensable step, the "final result" enabling manual plotting of a function's curve. Attempting to construct a curve without a complete variation table was risky and doomed to failure.

The arrival of technology reversed this logic. Today, the curve has become an immediate "given," accessible with a simple click. Curve plotting has disappeared from exams. Yet variation tables haven't vanished; their role has transformed. They now serve to **synthesize and give meaning** to an already-visible object whose veracity is not questioned (the calculator "does the work").

Theoretical Insight: The Drawing/Figure Distinction

Colette and Jean-Marie Laborde (Grenoble), creators of Cabri-géomètre in 1988, established a fundamental distinction: the *drawing* (what you see on screen) is not the *figure* (the abstract mathematical object). A construction is "robust" if it preserves its properties when elements are moved. This distinction prefigures what we must now make between the *raw AI response* and the *validated mathematical reasoning*.

1.2 What AI Changes Fundamentally

Generative AI pushes this logic to its extreme. It doesn't simply provide graphs or calculations: it offers **complete reasoning chains**, structured proofs, argued solutions.

This evolution aligns with the theoretical framework of **Didactical Situations Theory** developed by Guy Brousseau. In this framework, the "milieu" designates the antagonistic system the student confronts. Traditionally, this milieu consisted of the mathematical problem itself. With AI, the milieu gains a new element: *the machine-generated answer, which must be analyzed, validated, or refuted*.

Key Concept: Intelligence of Calculation

Luc Trouche (ENS Lyon) developed the concept of "intelligence of calculation": the challenge is no longer knowing how to compute by hand, but *knowing how to orchestrate paper-and-pencil with calculator to understand mathematics*. This concept now transfers: "AI intelligence" consists of knowing when and how to use the tool, and especially knowing how to evaluate what it produces.

2. From "Sherlock Mode" to "Columbo Mode": A Paradigm Shift

2.1 The Traditional Model: The Student-Investigator

Historically, students were placed in a "Sherlock Holmes" or Hercule Poirot posture. They started from an enigma—the problem statement—to search alone for the solution. The teacher was then the "revealer of enigmas" and the primary source of explanations.

This posture valorized the capacity to discover, produce, and solve. This model proved itself for decades. It developed autonomy, perseverance, mathematical creativity. But it rested on a fundamental asymmetry: the student didn't know, the teacher knew, and knowledge was transmitted through a process of guided discovery.

Theoretical Foundation: The Instrumental Approach

Pierre Rabardel (Paris 8, 1995) established a fundamental distinction: an *artifact* (the raw technical object) only becomes an *instrument* when the user develops *usage schemes*. This "instrumental genesis" comprises two movements: **instrumentation** (the tool transforms the user) and **instrumentalization** (the user adapts the tool). Without this genesis, AI remains a passive artifact.

2.2 The New Model: The Student-Validator

AI radically changes the situation by providing immediate access to knowledge and solutions. Faced with an answer already known or instantly generated, students must adopt a new posture: that of Lieutenant Columbo.

In the TV series, Columbo generally knows the culprit from the start of the episode. His job is not to discover "whodunit," but to *build the proof, track inconsistencies, and demonstrate guilt*. This is precisely the stance we must now cultivate in our students.

> ***The pedagogical challenge thus shifts from raw production to critical analysis and validation of machine-proposed solutions.***

Key Concept: Devolution Revisited

In Guy Brousseau's Didactical Situations Theory (Felix-Klein Medal 2003), devolution designates the process by which the teacher makes the student accept responsibility for solving the problem. With AI, this devolution takes on new meaning: the teacher must make the student accept responsibility for *validating* the answer, not just producing it.

2.3 Skills of "Columbo Mode"

Recent international literature identifies several key competencies valorized by this new paradigm. These competencies correspond to the upper levels of Bloom's taxonomy—analyze, evaluate, create—traditionally reserved for the most advanced students:

→ **Critical Analysis:** examine a proposed solution, identify its strengths and weaknesses, spot reasoning errors or AI "hallucinations" (fabricated information presented as fact).

→ **Regulation:** adjust one's own approach based on machine feedback, reformulate questions, clarify expectations.

→ **Validation:** verify mathematical coherence of an answer, confront it with other sources, test its limits.

→ **Evaluation:** judge the relevance of a strategy, appreciate the diversity of possible approaches, choose the most appropriate for the context.

→ **Metacognition:** reflect on one's own learning process, understand what one understands and what remains unclear.

Expert Strategy: Lateral Reading

Sam Wineburg (Stanford) compared how historians, students, and professional fact-checkers evaluate sources. Surprising result: fact-checkers surpass academics. Their secret? **Lateral reading**: quickly leave the source, open other tabs, search what others say about this source. This strategy directly transfers to AI: *never accept an answer without triangulation*.

Evaluation Grid: Universal Intellectual Standards

Richard Paul & Linda Elder (Foundation for Critical Thinking) identified standards applicable to any evaluation of reasoning. Facing an AI response, students can ask: Is it **clear** (comprehensible without ambiguity)? **Precise** (sufficiently detailed)? **Accurate** (verifiable, true)? **Relevant** (related to the question)? **Deep** (does it address complexities)?

3. The Teacher's Metamorphosis

3.1 From Transmitter to Orchestrator

In this new environment, teachers don't disappear—quite the contrary. But their role transforms profoundly. They become what some researchers call a "director of learning environments." This metaphor is illuminating: like a stage director, the teacher doesn't perform on stage themselves, but organizes the space, chooses angles, guides actors, gives meaning to the whole.

Learning no longer ends where AI gives an answer. It begins precisely at that point.

The teacher becomes the one who transforms an answer that's "too perfect" into an object to dissect, an explanation that's "too fluid" into material to question, an AI proposal into training ground for discernment.

Key Concept: Instrumental Orchestration

Luc Trouche introduced the concept of *instrumental orchestration* to describe the didactic management of available artifacts. An orchestration comprises **didactic configurations** (arrangement of tools and students) and **modes of exploitation** (ways to use them). Teachers integrating AI must design new orchestrations where AI plays a defined role in the pedagogical scenario.

3.2 Three New Axes of the Profession

UNESCO and the French Ministry of Education's work identifies three major axes around which the teacher's role now articulates:

First Axis: Curator and Validator

Since AI easily generates materials (flashcards, quizzes, summaries, exercises), teachers must select and guarantee the rigor of these contents. They no longer create everything ex nihilo, but filter, adjust, contextualize. This task demands reinforced disciplinary expertise, as one must be able to spot the subtle errors AI can produce.

Second Axis: Guide to Understanding

Teachers no longer just deliver methods, but help students make meaning, contextualize, connect notions together. Faced with AI that can provide procedures, teachers bring what no machine can offer: deep understanding, mathematical intuition, connection to lived experience.

Third Axis: Guardian of Critical Thinking

Their fundamental mission is to train students to unmask AI "hallucinations" or reasoning errors. Error then becomes a learning lever, a pedagogical opportunity. This is perhaps the most profound change: the machine's error, far from being an obstacle, becomes prime didactic material.

Key Concept: Double Genesis

Michèle Artigue (Felix-Klein Medal 2013) highlighted the double genesis necessary for technological integration: teachers must first master the tool for themselves (*personal genesis*), then learn to use it for teaching (*professional genesis*). This double requirement explains why "successful large-scale technological integration remains a major problem."

3.3 What Teachers Keep Exclusively

Certain competencies remain irreducibly human and constitute the core of teaching in the AI era:

- **Presence:** the way of entering the room, observing a furrowed brow that says "I no longer understand," alternating presence at the board and in the room. No AI "senses" a classroom.
- **Relationship:** the climate of trust that makes a student dare to ask a question, the reassuring smile, the voice adjustment when attention drifts.
- **Intelligence of the moment:** the strategy change decided in a second, the improvised phrase that unblocks an explanation, the activity invented because the class needs a detour.

Research Alert: Myths of Digital Technology

André Tricot & Franck Amadieu (Montpellier/Toulouse) deconstructed 11 myths about educational technology. Three are particularly relevant here:

- **The "digital natives" myth:** no scientific foundation. Young people are not naturally competent with digital tools for learning.
- **The motivation myth:** the link technology → motivation → learning is not established.
- **The autonomy myth:** autonomy is a prerequisite, not a result of digital use.

Key message: "Between resources and learning, there is an important person—the teacher—an important institution—the school—and very particular knowledge—school subjects."

4. The Student in Critical Dialogue: A New Learning Dynamic

4.1 AI as "Milieu" in Brousseau's Sense

Far from rendering students passive, AI reconfigures classroom situations by creating new didactic "milieus." In Didactical Situations Theory, the milieu is the antagonistic system that reacts to student proposals. It must offer feedback allowing students to adjust their strategy without direct teacher intervention.

AI can play this role in an unprecedented way. It proposes answers students must evaluate. It may produce errors, thereby creating learning opportunities. It can be queried iteratively, enabling dialogue resembling that of a patient tutor—*but a tutor whose statements must be systematically verified*.

Epistemological Alert: The Risk of "Ready-Made Answers"

Christian Orange (ENS, Free University of Brussels) argues that schools too often teach science's *results* rather than the *problems* that engendered them. AI amplifies this risk: it gives "answers" without showing underlying problems. Students obtaining an AI explanation risk considering it a "fact" rather than a *problematic construction to question*.

4.2 Constructive Conversation with the Machine

Recent research shows students are invited to "converse" with the machine, not to obtain a passive answer, but to construct mathematical meaning through reflective use. This conversation takes several forms:

- **Progressive questioning:** refine requests, specify constraints, reformulate to obtain a more pertinent answer.
- **Confrontation:** propose a counterexample, demand justification, test response limits.
- **Cross-verification:** compare AI's answer with other sources, with one's own knowledge, with peers' opinions.
- **Meta-reflection:** analyze why AI responded thus, what data it probably used, what biases might affect it.

Key Concept: Community of Inquiry

Matthew Lipman (Columbia), founder of philosophy for children, developed the concept of *community of inquiry*: a group that dialogues, questions, explores together. In this perspective, AI is not an oracle giving "THE" answer, but a **participant in dialogue** that must be questioned. Lipman's "4Cs" enrich Columbo competencies: Critical + Creative + Collaborative + Caring thinking.

4.3 From Reproducer to Judge

Students are no longer those who mechanically reproduce a learned approach, but those who judge the pertinence and limits of reasoning produced by a third party. This evolution is considerable. It valorizes high-level competencies —analysis, synthesis, evaluation—traditionally reserved for the most advanced students.

Bloom's taxonomy takes on full meaning here: lower levels (remember, understand, apply) can be partially delegated to AI, while **upper levels (analyze, evaluate, create) become the core of students' intellectual activity**.

Key Concept: Students' "Already-There"

Marie-Claude Penloup (Rouen) showed that students arrive in class with an "already-there" of writing practices. This concept applies to digital technology: students' digital writing practices (SMS, social media) constitute a support point. But caution: **no automatic transfer** of domestic competencies to school competencies. Just because a student uses TikTok doesn't mean they know how to use AI intelligently for homework.

5. Concrete Applications for Mathematics Class

Gradually, teachers are taking up AI and wanting to invest it in their teaching. The following proposals are merely avenues to explore, informed by researchers' work.

5.1 Scenario 1: Error Hunting

The teacher asks AI to solve a geometry problem. The generated solution contains deliberately (or fortuitously) subtle errors: a hypothesis used without verification, confusion between necessary and sufficient condition, an untreated special case.

Students, in groups, must identify these errors, correct them, and justify their corrections. This activity develops critical thinking while consolidating mathematical knowledge. It reverses the traditional relationship: *it's no longer the teacher who corrects, but the student who corrects the "expert"*.

5.2 Scenario 2: Contradictory Debate

Faced with an optimization problem, ask two different AIs (or the same AI with different prompts) to propose a solution. Students must compare approaches, identify each one's strengths and weaknesses, and argue in favor of one or the other.

This activity develops mathematical argumentation capacity and shows there isn't always a single "right" answer, but approaches more or less adapted to the context.

5.3 Scenario 3: Reverse Engineering

The teacher presents a complete AI-generated solution and asks students to reconstruct the problem statement that could have led to this solution. This activity reverses the usual process and develops deep understanding of links between problem and solution.

5.4 Scenario 4: Iterative Improvement

Students pose a problem to AI, analyze the response, then reformulate their question to obtain a more precise or complete answer. This iterative process develops both "prompting" competency (knowing how to ask pertinent questions) and deep mathematical understanding.

Field Testimony: AI in Life Sciences

Mélanie Fenaert (Blaise Pascal High School, Orsay), coordinator of TraAM on AI in life sciences, states: "*AI is perfect for working on critical thinking and source verification*." Her practices: have texts or images generated on scientific notions, then have students critique them; create pedagogical chatbots co-constructed with students. Her message: "AI education is also a question of equality, so as not to widen the gap between those who master the codes and others."

6. Training Critical Thinking: The A.U.D.I.T. Protocol

To structure the learning of critical thinking when facing AI, we humbly propose the **A.U.D.I.T.** protocol, a mnemonic acronym that guides students in their analysis. This protocol builds on Paul & Elder's intellectual standards and Bowker & Buitrago Ciro's "MT Literacy" concept.

A — Analyze the response

What exactly does AI say? What method does it use? What hypotheses does it make implicitly? (Paul-Elder standard: clarity)

U — Use your knowledge

Is this coherent with what I know? Are there contradictions with the course, with my previous experiences? (Paul-Elder standard: accuracy)

D — Doubt always the obtained answers

Did AI make calculation, logic, or interpretation errors? Did it "hallucinate" information? (Wineburg's lateral reading)

I — Interrogate the limits

Is this answer complete? Are there untreated special cases? Abusive generalizations? (Paul-Elder standards: depth, breadth)

T — Test and verify

Can I verify the result by another means? A counterexample? Another approach? (Bowker's post-editing)

Theoretical Foundation: From MT Literacy to AI Literacy

Lynne Bowker & Jairo Buitrago Ciro (Ottawa, 2019) defined competencies for "reasoned" use of machine translation. Their six dimensions prefigure AI Literacy: understand functioning, know how to use for reading/producing, appreciate ethical implications, evaluate if a task is adapted to the tool, pre-edit (formulate request well), post-edit (correct output).

7. Pitfalls to Avoid

7.1 The Risk of Passivity

The first pitfall would be letting students become passive consumers of ready-made answers. Research shows that unguided use of AI can reduce motivation to develop independent thinking. Teachers must therefore structure activities so that AI use is always accompanied by a critical analysis requirement.

7.2 Blind Trust

Generative AIs produce answers with assurance that can deceive. They sometimes "hallucinate," inventing nonexistent theorems or false but plausible proofs. Training in **constructive mistrust** is essential.

Illustration: The Enthoven vs. ChatGPT Test (June 2023)

At the 2023 philosophy baccalauréat (high school exit exam), a "match" opposed **Raphaël Enthoven** to **ChatGPT** on the subject "Is happiness a matter of reason?" Result: Enthoven 20/20, ChatGPT 11/20.

Graders identified AI's weaknesses: absence of problematic, hollow phrases, superficial citations, no commitment, evasive conclusion. Lev Fraenckel summarizes: "*This isn't philosophy—it's not stringing together beautiful phrases.*"

7.3 Abandoning Fundamentals

AI doesn't dispense with acquiring basics. A student who doesn't master fundamental notions will be incapable of evaluating the pertinence of a generated answer. **The foundational knowledge remains essential, as it underpins critical thinking.**

Insight: The Usage "Contract" Must Be Explicit

Sophie Soury-Lavergne (INSPE Grenoble) showed that displacement in dynamic geometry—a central functionality of Cabri—is not a natural reflex among students: it must be explicitly taught. Her Cnesco survey (2020) reveals that 60% of teachers never or rarely draw attention to displacement. Similarly, critical use of AI doesn't go without saying: an explicit "AI contract" is needed.

7.4 Ethical and Legal Questions

The **AI Usage Framework in Education** recalls the importance of protecting students' personal data. Using AI tools in class must respect this framework to guarantee confidentiality of information shared with these systems.

Conclusion: Toward a Pedagogy of Judgment

Current didactic evolution invites us to shift from a *school of resolution* to a *school of validation*. By adopting this "Columbo" posture, teaching gains depth: we no longer only ask students to know how to do, we ask them to understand what is done and to be able to prove it.

This transformation is not a threat to our profession—it is perhaps its finest opportunity. It frees us from the most mechanical tasks to refocus us on what constitutes the essence of teaching: **giving meaning, awakening critical thinking, accompanying the construction of autonomous thought**.

Thus technology, instead of impoverishing reflection, becomes the catalyst for renewed intellectual rigor. Lieutenant Columbo, with his falsely naive air and destabilizing questions, shows us the way: it's not the one who holds the answer who wins, but the one who knows how to question it.

"Just one more little thing...": This cult phrase of Columbo could become our pedagogical motto. Because it's always in this "last thing," in this additional questioning, that true learning nestles.

Synthesis: What Researchers Teach Us

1. **The tool alone is not enough** (Rabardel, Artigue, Tricot & Amadieu): without instrumental genesis, AI remains a passive artifact.

2. **Critical use must be taught** (Laborde, Soury-Lavergne): as with dynamic geometry, an explicit "contract" is needed.

3. **Lateral reading is the experts' strategy** (Wineburg): never remain prisoner of a single source.

4. **AI is a participant in dialogue, not an oracle** (Lipman): the community of inquiry integrates AI as an interlocutor to question.

5. **Students shift from Sherlock to Columbo** (Brousseau, Orange): from answer-seeker to critical validator.

Bibliographic References

Fundamental Didactic References

Artigue, M. (1989). Ingénierie didactique. *Recherches en Didactique des Mathématiques*, 9(3), 281-308.

Brousseau, G. (1998). *Théorie des situations didactiques*. La Pensée Sauvage.

Laborde, C. & Capponi, B. (1994). Cabri-géomètre constituant d'un milieu pour l'apprentissage de la notion de figure géométrique. *Recherches en Didactique des Mathématiques*, 14(1-2), 165-210.

Rabardel, P. (1995). *Les hommes et les technologies: approche cognitive des instruments contemporains*. Armand Colin.

Trouche, L. (2005). Construction et conduite des instruments dans les apprentissages mathématiques. *Recherches en Didactique des Mathématiques*, 25(1), 91-138.

References on Critical Thinking and Information Evaluation

Lipman, M. (2003). *Thinking in Education* (2nd ed.). Cambridge University Press.

Paul, R. & Elder, L. (2021). *Critical Thinking: Tools for Taking Charge of Your Learning and Your Life* (4th ed.). Rowman & Littlefield.

Wineburg, S. & McGrew, S. (2019). Lateral Reading and the Nature of Expertise. *Teachers College Record*, 121(11).

References on Educational Technology

Bourdais, A. (2021). *Traducteurs en ligne et enseignement-apprentissage de l'anglais* (Doctoral thesis). Université de Lyon.

Bowker, L. & Buitrago Ciro, J. (2019). *Machine Translation and Global Research*. Emerald Publishing.

Orange, C. (2012). *Enseigner les sciences: Problèmes, débats et savoirs scientifiques en classe*. De Boeck.

Penloup, M.-C. (1999). *L'écriture extrascolaire des collégiens*. ESF.

Plane, S. (1996). Écriture, réécriture et traitement de texte. In *L'apprentissage de l'écriture de l'école au collège*. PUF.

Soury-Lavergne, S. (2020). La géométrie dynamique pour l'apprentissage et l'enseignement des mathématiques. *Rapport Cnesco*.

Tricot, A. & Amadieu, F. (2020). *Apprendre avec le numérique: mythes et réalités* (2nd ed.). Retz.

References on AI in Education

Emprin, F. & Richard, P.R. (2023). Intelligence artificielle et didactique des mathématiques. *Annales de Didactique et de Sciences Cognitives*, 28, 131-181.

Kasneci, E. et al. (2023). ChatGPT for good? On opportunities and challenges of large language models for education. *Learning and Individual Differences*, 103, 102274.

Ministère de l'Éducation nationale (2024). *Intelligence artificielle et éducation: apports de la recherche et enjeux pour les politiques publiques*.

UNESCO (2024). *Référentiels de compétences en IA pour les élèves et les enseignants*. Paris: UNESCO.

Author: Philippe Dupeyrat, Mathematics Education Inspector, France
Context: Classroom-tested and refined through pilot implementations. Developed for French secondary education (middle school grades 6-8, high school grades 9-12, ages 12-18), adaptable internationally
Full Resources: https://philipped79.github.io/audit-ia (French)

This methodology represents a fundamental rethinking of mathematics education for the AI era—from a "Sherlock Holmes" model (students as solution-seekers) to a "Columbo" model (students as critical validators). The pedagogical challenge is no longer primarily about finding answers, but about intelligently evaluating them.